Two-Factor Repeated Measures Designs

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  1. There can be more than one repeated measure, or *within-subjects* factor.
  2. In a repeated measures design, there can be more than one group of subjects, in which case we have a *between-subjects* factor. Indeed, there can be several between-subjects factors combined factorially.

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- The extensions can go in several directions:
    1. There can be more than one repeated measure, or *within-subjects* factor.
    2. In a repeated measures design, there can be more than one group of subjects, in which case we have a *between-subjects* factor. Indeed, there can be several between-subjects factors combined factorially.
    3. When within-subjects and between-subjects factors occur in the same design, we can refer to the design as a *between-within* design.

The $S \times A \times B$ Within-Subjects Design

Introduction

- We can extend the 1-Way Repeated Measures design to two or more repeated measures factors, combined factorially.
- An example is presented in RDASA3 Section 15.2.
- Each of 6 subjects were presented with figures that were varied across 3 levels of *Distortion* (Factor A) and 3 levels of *Orientation* (Factor B). So each subject was presented with 9 photos in all. The order of presentation was varied randomly for each subject.
- Data are presented in Table 15.3, and are available online in the file *Table1503.csv*.

The $S \times A \times B$ Within-Subjects Design

Introduction

Table 15.3 Data for a two-factor repeated-measures experiment (A is orientation, B is distortion)

	B_1			B_2			B_3		
Subjects	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
1	1.18	2.40	2.48	4.76	4.93	3.13	5.56	4.93	5.21
2	1.14	1.55	1.25	4.81	4.73	3.89	4.85	5.43	4.89
3	1.02	1.25	1.30	4.98	3.85	3.05	4.28	5.64	6.49
4	1.05	1.63	1.84	4.91	5.21	2.95	5.13	5.52	5.69
5	1.81	1.65	1.01	5.01	4.18	3.51	4.90	5.18	5.52
6	1.69	1.67	1.04	5.65	4.56	3.94	4.12	5.76	4.99

Cell and marginal means

	A_1	A_2	A_3	$\bar{Y}_{..k}$
B_1	1.315	1.692	1.487	1.498
B_2	5.020	4.577	3.412	4.336
B_3	4.807	5.410	5.465	5.227
$\bar{Y}_{.j.}$	3.714	3.89	3.455	$\bar{Y} = 3.687$

The $S \times A \times B$ Within-Subjects Design

Introduction

- As is usually the case with repeated measures data, we need to recast the data prior to analysis. We start by reading in the data and adding a Subject variable.

```
> Table1503 <- read.csv("Table1503.csv")
> Subject <- 1:6
> Table1503 <- cbind(Subject, Table1503)
> Table1503

  Subject A1B1 A2B1 A3B1 A1B2 A2B2 A3B2 A1B3 A2B3 A3B3
1       1 1.18 2.40 2.48 4.76 4.93 3.13 5.56 4.93 5.21
2       2 1.14 1.55 1.25 4.81 4.73 3.89 4.85 5.43 4.89
3       3 1.02 1.25 1.30 4.98 3.85 3.05 4.28 5.64 6.49
4       4 1.05 1.63 1.84 4.91 5.21 2.95 5.13 5.52 5.69
5       5 1.81 1.65 1.01 5.01 4.18 3.51 4.90 5.18 5.52
6       6 1.69 1.67 1.04 5.65 4.56 3.94 4.12 5.76 4.99
```

The $S \times A \times B$ Within-Subjects Design

Melting the Data

- Next, we "melt" the data, using the `melt` function from the `reshape` library.
- Note how, in the call, I just use the numbers of the variables to be used as id.vars and measured.vars.

```
> temp <- melt(Table1503, id.vars = 1, measure.vars = 2:10)
> temp[1:10, ]

   Subject variable value
1        1     A1B1  1.18
2        2     A1B1  1.14
3        3     A1B1  1.02
4        4     A1B1  1.05
5        5     A1B1  1.81
6        6     A1B1  1.69
7        1     A2B1  2.40
8        2     A2B1  1.55
9        3     A2B1  1.25
10       4     A2B1  1.63
```

- Taking a look at this, we see that R still has no way of knowing how to identify which columns stand for which factors, and which levels are involved.
- Thorough study of the `reshape` package and the (different) `reshape` function in R may well enable you to automate the further processing of the data. In this case, I simply create new variables to tell R which observations are at which levels of each factor.
- Code on the next slide shows how I did this.

The $S \times A \times B$ Within-Subjects Design

Reshaping the Data

```
> A <- rep(c(rep(1, 6), rep(2, 6), rep(3, 6)), 3)
> B <- c(rep(1, 18), rep(2, 18), rep(3, 18))
> rm.data <- data.frame(cbind(temp$Subject, temp$value, A, B))
> colnames(rm.data) <- c("Subject", "Rating", "Orientation", "Distortion")
> rm.data$Subject <- factor(rm.data$Subject)
> rm.data$Orientation <- factor(rm.data$Orientation)
> rm.data$Distortion <- factor(rm.data$Distortion)
```

- Now that the data are properly arranged for analysis, a simple call to the `ezANOVA` function in the ez library accomplishes the analysis.
- You can verify that this analysis agrees with Table 15.4 in RDASA3.

The $S \times A \times B$ Within-Subjects Design

Analyzing with ezANOVA

```
> ezANOVA(rm.data, wid = .(Subject), dv = .(Rating), within = .(Orientation, Distortion))

$ANOVA
                  Effect DFn DFd          F            p p<.05       ges
2            Orientation   2  10   9.233704 5.348849e-03     * 0.1586956
3             Distortion   2  10 302.559959 1.135536e-09     * 0.9364271
4 Orientation:Distortion   4  20   7.750117 6.088323e-04     * 0.4800754

$`Mauchly's Test for Sphericity`
                  Effect          W         p p<.05
2            Orientation 0.96186281 0.9251801
3             Distortion 0.92723232 0.8597598
4 Orientation:Distortion 0.05631215 0.4226226

$`Sphericity Corrections`
                  Effect       GGe        p[GG] p[GG]<.05       HFe
2            Orientation 0.9632638 6.038310e-03         * 1.5551085
3             Distortion 0.9321683 3.939469e-09         * 1.4647649
4 Orientation:Distortion 0.4617494 1.149319e-02         * 0.7201061
         p[HF] p[HF]<.05
2 5.348849e-03         *
3 1.135536e-09         *
4 2.753985e-03         *
```

The $S \times A \times B$ Within-Subjects Design

Analyzing with ezANOVA

Table 15.4 ANOVA of the data of Table 15.3

					Significance		
Source	*df*	*SS*	*MS*	*F*	*p*	*G–G*	*H–F*
S	5	.544	.109				
A (orientation)	2	1.749	.874	9.23	.005	.006	.005
SA	10	.947	.095				
B (distortion)	2	136.554	68.277	302.56	.000	.000	.000
SB	10	2.257	.226				
AB	4	8.560	2.140	7.75	.001	.011	.003
SAB	20	5.522	.276				
Total	53	156.133					

Note: G–G refers to the Greenhouse–Geiser correction for nonsphericity and *H–F* refers to the Huynh–Feldt correction.

The $S \times A \times B$ Between-Within Design

Introduction

- A frequently-used design is the $S \times A \times B$ between-within design, with the A and B fixed effects factors crossed factorially, but with different groups of subjects in each cell representing the levels of the A factor, but each subject being measured on all levels of the B factor.

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- MWL refer to it as a "mixed" design, a poor choice because it is easily confused with a "mixed model" (meaning random effects and fixed effects in the same design. I'll stick to the term "between-within" when describing such models.

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- An example of data for such a design is shown in RDASA3, Figure 15.5.

The $S \times A \times B$ Between-Within Design

Introduction

Table 15.5 Data for a design with one between-subjects (*A*) and one within-subjects (*B*) factor

Method of instruction		Time of test B_1	B_2	B_3	B_4	$\bar{Y}_{ij.}$
A_1	S_{11}	82	48	41	53	56
	S_{21}	72	70	51	45	62
	S_{31}	43	35	30	12	30
	S_{41}	77	41	61	31	50
	S_{51}	43	43	21	29	34
	S_{61}	67	39	30	40	44
	$\bar{Y}_{.1k}$	64	46	39	35	$\bar{Y}_{.1.} = 46$
A_2	S_{12}	71	53	50	62	59
	S_{22}	89	67	76	68	75
	S_{32}	82	84	83	71	80
	S_{42}	56	56	55	45	53
	S_{52}	64	44	44	52	51
	S_{62}	76	74	64	74	72
	$\bar{Y}_{.2k}$	73	63	62	62	$\bar{Y}_{.2.} = 65$
A_3	S_{13}	84	80	75	77	79
	S_{23}	84	72	63	81	75
	S_{33}	76	54	57	61	62
	S_{43}	84	66	61	77	72
	S_{53}	67	69	55	69	65
	S_{63}	61	67	55	61	61
	$\bar{Y}_{.3k}$	76	68	61	71	$\bar{Y}_{.3.} = 69$
	$\bar{Y}_{..k}$	71	59	54	56	$\bar{Y}_{...} = 60$

The $S \times A \times B$ Between-Within Design

Reading and Reshaping the Data

```
> Table1505 <- read.csv("Table1505.csv")
> rm.data <- data.frame(melt(Table1505, id.vars = 1:2, measure.vars = 3:6))
> rm.data$A <- factor(rm.data$A)
> rm.data$Subject <- factor(rm.data$Subject)
> colnames(rm.data) = c("Subject", "Method", "Time", "Score")
> rm.data[1:12, ]

   Subject Method Time Score
1        1      1   B1    82
2        2      1   B1    72
3        3      1   B1    43
4        4      1   B1    77
5        5      1   B1    43
6        6      1   B1    67
7        7      2   B1    71
8        8      2   B1    89
9        9      2   B1    82
10      10      2   B1    56
11      11      2   B1    64
12      12      2   B1    76
```

The $S \times A \times B$ Between-Within Design

Analyzing the Data with ezANOVA

```
> ezANOVA(rm.data, wid = .(Subject), dv = .(Score), between = .(Method), within = .(Time))

$ANOVA
       Effect DFn DFd         F            p p<.05        ges
2      Method   2  15  7.754636 4.869065e-03     * 0.43225191
3        Time   3  45 18.717131 5.017839e-08     * 0.24754979
4 Method:Time   6  45  3.155378 1.142322e-02     * 0.09984871

$`Mauchly's Test for Sphericity`
       Effect          W            p p<.05
3        Time 0.08021159 1.958756e-06     *
4 Method:Time 0.08021159 1.958756e-06     *

$`Sphericity Corrections`
       Effect       GGe        p[GG] p[GG]<.05       HFe        p[HF]
3        Time 0.6777957 4.525655e-06         * 0.7849439 1.007433e-06
4 Method:Time 0.6777957 2.721846e-02         * 0.7849439 2.032304e-02
  p[HF]<.05
3         *
4         *
```

Table 15.8 ANOVA of the data in Table 15.5

Source	*df*	Sum of squares	Mean square	*F*	*p*	*G–G*	*H–F*
A (Method)	2	7,248	3,624.00	7.76	0.005		
S/A	15	7,010	467.33				
B (Time)	3	3,132	1,044.00	18.72	0.000	0.000	0.000
AB	6	1,056	176.00	3.16	0.011	0.027	0.015
$B \times S/A$	45	2,510	55.78				